Fiber length estimation method for beamforming at millimeter wave band RoF-FWA system

Mizuki Suga, Kota Ito, Yushi Shirato, Naoki Kita, and Takeshi Onizawa

NTT Access Network Service Systems Laboratories, NTT Corporation,
1–1 Hikarinooka, Yokosuka-Shi, Kanagawa 239–0847, Japan
a) mizuki.suga.tx@hco.ntt.co.jp

Abstract: To supplant partial optical access networks, we study a large capacity transmission system with millimeter wave applied radio over fiber (RoF). In this system, it is desirable from the viewpoints of downsizing and power saving that the base station (BS) is simplified and the central station (CS) controls beamforming. However, this demands fiber length estimation because each wavelength must be given a different phase rotation due to chromatic dispersion in the optical fiber. This paper proposes a method to estimate fiber length from CS to BS supporting wireless terminal (WT) by utilizing time synchronization; its performance is evaluated.

Keywords: RoF, millimeter wave, beamforming

Classification: Wireless Communication Technologies

References
1 Introduction

Applying a large capacity communication system that uses millimeter wave links as an alternative to partial optical networks is drawn attention [1]. Existing millimeter wave fixed wireless access (FWA) systems suffer low efficiency because the high propagation loss constrains the communication area. Our solution is a novel FWA system that uses the radio over fiber (RoF) technique; we call it millimeter wave band RoF-FWA system. This system sets the signal processing function and the RF processing function in the central station (CS) and base station (BS), respectively. Larger communication coverage is realized by setting multiple BSs that are connected to one CS via a passive optical network (PON). This provides significant installation advantages as each BS is expected downsizing and power saving by simplifying.

Millimeter wave band RoF-FWA system requires beamforming to acquire link budget. In order to perform beamforming while simplifying all BSs, this system executes beam control in CS [2, 3]. When CS controls a beam, a different wavelength is allocated to each antenna element to secure the phases appropriate. However, each wavelength experiences a different phase rotation due to chromatic dispersion in the optical fiber, and this deviation must be cancelled for appropriate beamforming. Since the phase rotation is determined by wavelength and fiber length, fiber length estimation is necessary.

Two fiber length estimation methods are common: using transmission time obtained by optical time domain reflectometer (OTDR) [4] or point to multipoint (P2MP) discovery [5]. Reference [4] estimates the fiber length from the time taken for a pulse signal transmitted by CS is reflecting back by BS. However, multiple BSs connect to one CS as this system, making difficult to apply this method because discerning which BS the signal is reflecting from becomes impossible. Reference [5] estimates the fiber length by using P2MP discovery to measure the round trip time (RTT). This method is not suitable for millimeter wave band RoF-FWA system as each BS is made more complicated by the addition of a function that transmits a frame with embedded identifier to CS. Furthermore, even if the fiber length of each BS is obtained by these methods, it is unclear which BS should
support the wireless terminal (WT). Therefore existing methods are not suitable for this system.

We propose a novel fiber length estimation method that utilizes the communication time difference created by two wavelengths obtained by time synchronization. This method can estimate the fiber length from CS to the BS that is supporting the target WT. Since the fiber length estimates include the error imposed by the time synchronization error, this paper clarifies the wavelength setting that minimizes the estimation error. In addition, beamforming characteristics achieved with estimated fiber length are evaluated, and the influence of the time synchronization error on beamforming performance is verified by simulations.

2 Proposed fiber length estimation method

The structure of proposed method in millimeter wave band RoF-FWA system shown in Fig. 1. It assumes the use of broad beams that are used to establish a low speed mode for transmitting control signals. CS and WT are time synchronized, downlink and link used different wavelengths. First, CS transmits a training signal from one BS antenna element; WT receives the signal using one WT antenna element. At this point, CS and WT obtain transmission start timing $t_{\text{d,tx}}$ and reception start timing $t_{\text{d,rx}}$, respectively. Next, WT returns training signal and CS receives this signal using the same antenna elements used for downlink communication. At this point WT and CS obtain transmission start timing $t_{\text{u,tx}}$ and reception start timing $t_{\text{u,rx}}$, respectively. WT embeds $t_{\text{d,rx}}$ and $t_{\text{u,tx}}$ in the returning signal, and CS estimates the fiber length from this information. CS obtains downlink and uplink total communication duration, $t_d$ and $t_u$, from transmission and reception start times.

\[
\begin{align*}
t_d &= t_{\text{d,rx}} - t_{\text{d,tx}} \\
t_u &= t_{\text{u,rx}} - t_{\text{u,tx}}
\end{align*}
\]

Total communication durations can also be given by Eq. (2),

\[
\begin{align*}
t_d &= t_{\text{fd}} + t_r + t_p \\
t_u &= t_{\text{fu}} + t_r + t_p
\end{align*}
\]

where, $t_{\text{fd}}$ and $t_{\text{fu}}$ are fiber transmission time in downlink and uplink, $t_r$ is wireless transmission time, $t_p$ is total signal processing time at CS, BS, and WT. When calculates the difference between $t_d$ and $t_u$, $t_r$ and $t_p$ cancel out and the difference of $t_{\text{fd}}$ and $t_{\text{fu}}$ remains as shown in Eq. (3).

\[
t_d - t_u = t_{\text{fd}} - t_{\text{fu}}
\]

In here, fiber transmission time can be obtained by fiber length $l$ and group delay time per distance in downlink and uplink, $\tau_d$, $\tau_u$. It is a known parameter determined by the wavelength used for fiber transmission and the chromatic dispersion of the fiber.

\[
\begin{align*}
t_{\text{fd}} &= l \cdot \tau_d \\
t_{\text{fu}} &= l \cdot \tau_u
\end{align*}
\]

Plugging Eq. (4) into Eq. (3) yields:

\[
t_d - t_u = l \cdot \tau_d - l \cdot \tau_u.
\]
Solving Eq. (5) for \( l \) yields:

\[
I = \frac{t_d - t_u}{\tau_d - \tau_u}
\]

(6)

As shown above, the fiber length can be estimated from measured values and known parameters.

3 Performance evaluations

Actual total communication durations \( t_d \) and \( t_u \) include measurement error due to the time synchronization error. Therefore, this section confirms which wavelength setting minify the fiber length estimation error and verifies the influence of the time synchronization error on beamforming performance.

3.1 Beamforming scheme

The beamforming scheme used this evaluation directly connects a unique wavelength to each BS antenna element. The phase for beamforming and the added phase rotation created by the chromatic dispersion of \( i \)-th (\( 1 \leq i \leq n \)) antenna element are represented by \( \theta_i, \phi_i \), respectively; \( n \) is the number of antenna elements. The phase for beamforming and the added phase rotation are given by Eq. (7), (8)

\[
\theta_i = \frac{2\pi d_i \sin \psi}{\lambda_{RF}}
\]

(7)

\[
\phi_i = 2\pi \cdot f_{RF} \cdot l \cdot \tau_i,
\]

(8)

where \( d_i \) is distance from reference antenna element, \( \psi \) is signal arrival direction, \( \lambda_{RF} \) is RF wavelength, \( f_{RF} \) is RF frequency, \( \tau_i \) is the group delay time of \( \lambda_i \) (wavelength assigned to \( i \)-th antenna element). The phase of \( i \)-th antenna element set in the phase control unit (\( \theta_{iCS} \)) is shown in Eq. (9).

\[
\theta_{iCS} = \theta_i - \phi_i,
\]

(9)

The phase of signal arriving at BS (\( \theta_{iBS} \)) is added \( \phi_i \) in fiber as shown in Eq. (10).

\[
\theta_{iBS} = \theta_{iCS} + \phi_i,
\]

(10)

Thus, \( \phi_i \) is canceled by fiber transmission and only \( \theta_i \) remains, in fact \( \theta_{iBS} = \theta_i \).

3.2 Simulation

Simulation parameters are shown in Table I. RF frequency is 60 GHz band which is a typical millimeter wave band, and fiber length is 10 km (standard length in optical access networks). The fiber is single mode fiber (SMF), wavelength used fiber length estimation (\( \lambda_d, \lambda_u \)) is 1300–1625 nm, these values conform to recommendation ITU-T [6]. Wavelength of fiber transmission is 1300 nm band which is one of common wavelength band used in optical network systems. The allocated wavelengths have equal spacing (\( \Delta \lambda \)) of 0.2–1 nm. Time synchronization error (described below as time error) is 0.1–20 ns, it takes account of the accuracy of the global positioning system (GPS).

Fig. 2(a) shows the fiber length estimation error for the wavelengths used, for the case that time error is 1 ns. The four lines show the impact of wavelength difference on fiber length estimation (\( |\lambda_d - \lambda_u| \)). Since the wavelengths are limited to 1300–1625 nm, the plots become shorter as the wavelength difference increases.
This figure shows that the fiber length estimation error tends to shrink as the wavelength difference increases. This result is reasonable because large differences in wavelengths yield large differences in fiber transmission time, making the time error relatively small. In addition, since the chromatic dispersion increase yields large differences in fiber transmission time, the estimation accuracy improves with longer wavelengths for the same reason. The following evaluation uses wavelengths of 1300 nm and 1500 nm to replicate the wavelengths used in actual optical networks.

Fig. 2(b) shows the beam direction error, which is the deviation from the desired direction of beam, and the time error. The five lines plots the results gained when $\Delta \lambda$ allocated to eight antenna elements is varied in the range of 0.2–1 nm. It can be confirmed that beam direction accuracy deteriorates in proportion to time error regardless of $\Delta \lambda$. This is because the fiber length estimation error becomes large as time error increases. It is confirmed that the beam direction error is lower and the beam direction is accurate with $\Delta \lambda$ is narrower. Since the phase rotation offset is determined by the estimated fiber length in this beamforming scheme (explained in 3.1), the added phase rotation includes error due to fiber estimation error. This error increases with the group delay time as shown in Eq. (8). Therefore, if $\Delta \lambda$ becomes large and a longer wavelength is used, the error in phase rotation offset increases and beam direction accuracy is degraded. In this evaluation case, even a slight time error yields significant beam direction error that exceeds the half power beam width (HPBW) at $\Delta \lambda = 1$ nm. On the other hand, even if the time error is 20 ns, the beam direction error is less than HPBW at $\Delta \lambda = 0.4$ nm or less. 0.4 nm is about 70 GHz when converted into a frequency in the optical wavelength band; this spacing is practical if the 60 GHz band is used for RF communication.

Fig. 2(c) shows the beam pattern when $\Delta \lambda$ is 0.4 or 1 nm. In the case of $\Delta \lambda = 1$ nm, beam form becomes distorted and the peak level decreases as the time error increases. When the time error is 20 ns, the beamforming gain in the desired direction is reduced by about 12.7 dB compared to that without time error. However, in the case of $\Delta \lambda = 0.4$ nm, since the beam direction error is less than HPBW, the beamforming gain decrease in the desired direction is very low, about 1.5 dB, and beam form distortion around the main beam is negligible. The above results show that proposed method is an available way of achieving adequate beamforming gain in the desired direction with low degradation using practical parameter.

### Table 1. Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency</td>
<td>60 GHz band</td>
</tr>
<tr>
<td>BS antenna</td>
<td>8 antenna element linear array</td>
</tr>
<tr>
<td>Half wavelength spacing</td>
<td></td>
</tr>
<tr>
<td>Fiber type</td>
<td>SMF [6]</td>
</tr>
<tr>
<td>Fiber length</td>
<td>10 km</td>
</tr>
<tr>
<td>Wavelength used fiber length estimation $(\lambda_d, \lambda_u)$</td>
<td>1300–1625 nm</td>
</tr>
<tr>
<td>Wavelength of fiber transmission $(\lambda_i)$</td>
<td>1300 nm band</td>
</tr>
<tr>
<td>Wavelength spacing $(\Delta \lambda)$</td>
<td>0.2–1 nm</td>
</tr>
<tr>
<td>Time error</td>
<td>0.1–20 ns</td>
</tr>
</tbody>
</table>
4 Conclusion

We proposed a fiber length estimation method based on time synchronization for a millimeter wave band RoF-FWA system. This paper examined the wavelength setting to reduce the fiber length estimation error and evaluated the influence of time error on beamforming accuracy. Simulations showed that the fiber length estimation error tends to fall as the wavelength difference widens or longer wavelengths are used. The results showed that proposed method makes beamforming possible with high accuracy as the beam direction error can be reduced to under the HPBW and the beamforming gain degradation on the desired direction is about 1.5 dB or so in the case of practical wavelength intervals.

![Fig. 2. Simulation results](image)
Single-feed dual-band dual-polarized textile antenna

Daisuke Yamanaka\textsuperscript{1a}) and Masaharu Takahashi\textsuperscript{1b})

\textsuperscript{1} Graduate School of Engineering, Chiba University,
1–33 Yayoicho, Inage-ku, Chiba-shi, Chiba 263–8522, Japan
\textsuperscript{2} Center for Frontier Medical Engineering, Chiba University,
1–33 Yayoicho, Inage-ku, Chiba-shi, Chiba 263–8522, Japan
\textsuperscript{a}) 14t0844w@chiba-u.jp
\textsuperscript{b}) omei@faculty.chiba-u.jp

Abstract: In recent years, many small antennas used near a human body have been proposed in the medical field. Most of them are designed in consideration of the influence of a human body having high dielectric properties and assume use at a single frequency or a single polarization. We proposed a dual-polarized textile antenna for the 5.2 GHz band as our previous study.

In this paper, we report the design of a dual-band dual-polarized antenna using single point coplanar feeding. This antenna has excellent radiation patterns and reflection coefficients for each of the desired frequency.

Keywords: textile antenna, dual band, dual polarized, patch antenna

Classification: Antennas and Propagation

References


1 Introduction

In recent years, a large number of textile antennas have been studied in communication near the human body [1, 2, 3, 4, 5]. Since the human body has a high dielectric constant, it is necessary to consider loss and antenna characteristics. Also, in general, communication devices worn in the vicinity of the human body may influence reception ability or transmission ability due to the impact of movement or posture change.

In our previous research, we designed a 5.2 GHz textile antenna for biological information monitoring systems [6, 7]. This previous antenna can transmit biological information regardless of changes in a patient’s posture or motion by radiating elliptical polarization, which is ensured by the received power experiment. In a propagation environment of a patient’s room, motion and posture of a patient do not always move, and it can be imagined that a patient’s motion and posture are changed while maintaining a certain posture direction, so perfect circular polarization is not necessarily required.

However, if a receiving antenna is entirely orthogonal to this elliptical radiation pattern, a received power may be smaller than expected. Besides, there are individual differences in the electrical characteristics of the human body, and a single-frequency patch antenna with a narrow frequency bandwidth may cause a deviation from the desired frequency.

To solve these problems, we designed the two-frequency design of textile patch antenna with planar structure and coplanar feed by using a slit between a microstrip line and a radiation element and L-shaped slot on the patch based on the previously proposed antenna. An antenna with a design that radiates two orthogonal elliptical polarizations enables more stable communication because it is not more sensitive to differences in the electrical characteristics of the human body due to has two design frequencies. This antenna also is flexible in design to determine the frequency of interest. Furthermore, having multiple frequencies and polarizations contributes to more flexible system construction, such as polarization switching and diversity operation depending on a communication status.

2 Design for dual-band frequency

Fig. 1 shows a designed antenna and some parameters for characteristics evaluation, and this simulation was performed by time domain solver of CST MW-Studio 2019 [8]. This antenna consists of a patch and a ground plane made of conductive textile. A dielectric of this antenna is a felt, that relative permittivity was set to 1.36. We inserted a slit between feeding microstrip line and patch; this slit works to improve impedance matching at both desired frequencies. This slit length is 1 mm longer compared to the previous study [6].
Next, we designed an L-shaped slot on the patch to extend a current stream of a low-side resonance frequency. An antenna that resonates at two desired frequencies can be designed by adjusting the size of the radiation element, the length of the slit and the size of the L-shaped slot.

Fig. 2(a) shows the analysis results of the reflection coefficient when the slit length was fixed at 11 m, and the L-shaped slot is loaded on the patch. From the results, it is possible to change the resonance frequency $f_1$ on the low-frequency side by the length of the L-shaped slot without changing the resonance frequency $f_2$ on the high-frequency side as well, and the maximum width of dual resonant frequency was around 550 MHz when the length of $S_s$ slot set to 6.0 mm.

A surface current distribution is shown in Fig. 2(b-1) and (b-2). The current of lower-frequency is stretched by detouring by the L-shaped slot whereas the current of frequency $f_2$ is only slightly affected, and this current direction is opposite to high-frequency’s one. These frequencies are determined only by the slit and the L-shaped slot on the patch, and it can be confirmed that the frequency design is flexible.

Fig. 1. Designed antenna
3 Dual polarization for dual band frequency

Since this antenna radiates elliptically polarized waves as close as possible to linear polarization, it is not appropriate to evaluate from the viewpoint of a circularly polarized wave. Therefore, we used $E_\theta$ and $E_\phi$ for evaluation in this section.

Fig. 3(a), (b), (c) and (d) shows antenna gains of both of low resonant frequency $f_1$ and high resonant frequency $f_2$. These results provided us that obtained similar gains at the $f_1$ and the $f_2$ regardless of the size of the L-shaped slot, these gains are all around 8.42 dBi and 8.56 dBi respectively, and we
confirmed that this antenna could radiate dual-polarized wave on each frequency even though a sizeable L-shaped slot inserted to the specified position of the patch.

4 Conclusion

In this paper, we propose a new single-fed dual-band dual-polarization textile patch antenna with the slits and L-shaped slot inserted. This antenna structure can select the second frequency from the band in a range of around 550 MHz in addition to the main frequency and has good resonance characteristics and the antenna gain.

In terms of the polarization and the antenna gain, we obtained around 8.42 dBi for $f_1$ and 8.56 dBi for $f_2$ respectively when the $S_s$ length was set to 4.5 mm. In each low-side resonant frequency that depends on a length of its L-shaped slot, without significantly affecting the resonance and gain on another one.

Fig. 3. Antenna gains on each plane
Theoretical performance evaluation of MU-MIMO THP with user scheduling

Karen Taguchi, Ryota Mizutani, Yukiko Shimbo, Hirofumi Suganuma, and Fumiaki Maehara

Graduate School of Fundamental Science and Engineering, Waseda University, 3–4–1 Ohkubo, Shinjuku-ku, Tokyo 169–8555, Japan
a) fumiaki.m@waseda.jp

Abstract: This paper presents the theoretical system-level performance of multi-user multiple-input and multiple-output (MU-MIMO) Tomlinson-Harashima precoding (THP) with user scheduling. In our performance evaluation, proportional fairness (PF), which makes a reasonable compromise between fairness among users and the benefit of multi-user diversity, is implemented as a user scheduling technique, and the effect of modulo loss resulting from THP modulo operation at the receiver is taken into account using mod-Λ channel-based analysis, which provides accurate theoretical performance. Moreover, considering the application to the PF metric, the performance of the mod-Λ channel-based PF metric is compared with that of the traditional Shannon-Hartley theorem-based metric.

Keywords: MU-MIMO, Tomlinson-Harashima precoding (THP), system capacity, mod-Λ channel, proportional fairness (PF)

Classification: Wireless Communication Technologies

References

1 Introduction

With the rapid growth in the use of smart devices, the demand for mobile wireless services has increased exponentially, leading to expectations of higher speed, larger capacity, and lower latency. In 2022, the amount of wireless traffic is estimated to reach 71% of all IP traffic [1], and the fifth-generation mobile communication system (5G) is soon to be commercialized in terms of enhanced mobile broadband (eMBB), ahead of ultra-reliable and low-latency communications (URLLC) and massive machine type communications (mMTC). Multi-user multiple-input and multiple-output (MU-MIMO) is an essential technique for 5G systems because larger capacity can be realized via a single antenna mounted on a mobile station (MS) [2].

Precoding techniques are essential for implementation of MU-MIMO, and are categorized into two approaches: linear precoding (LP) and non-linear precoding (NLP). NLP provides better system capacity than LP because it reduces noise enhancement, and has thus emerged as a candidate technique for 5G systems [3, 4, 5]. Of the various NLP schemes, Tomlinson-Harashima precoding (THP) is considered a practical approach because the perturbation vector can be generated by a simple modulo operation [4, 6, 7].

From a system-level perspective, the combination of MU-MIMO with user scheduling must be taken into account [8, 9, 10]. This is because the capacity of
the entire system strongly depends on how simultaneous users are selected from existing users within a certain cell. Therefore, the performance evaluation is expected to consider the impact of user scheduling as well as MU-MIMO. Computer simulations are considered the surest ways to investigate system performance, but require large computational cost because wireless signal processing, such as modulation and demodulation, needs to be conducted. In this sense, theoretical analysis is considered a powerful tool because mathematical expressions enable us to comprehensively investigate the influence of system parameters on system performance without any time-consuming computer simulations [11].

Considering the above background, we investigate the theoretical system capacity of MU-MIMO THP in terms of user scheduling. The focus of our work is to account for the impact of the modulo loss peculiar to THP, which provides more accurate theoretical performance than our previous work [9] based on the Shannon-Hartley theorem-based approach [6, 7]. Moreover, although the authors in [10] considered the effect of modulo loss for performance evaluation, its effect is given as a constant margin degradation of 0.5 dB, which has left further room for improvement. Thus, in this paper, proportional fairness (PF) [8], which makes a reasonable compromise between fairness among users and the benefit of multi-user diversity, is considered as a user scheduling technique, and the theoretical system performance of MU-MIMO THP with PF is analyzed based on the mod-Λ channel [12]. Moreover, to clarify the required accuracy of the PF metric, the performance of the mod-Λ channel-based PF metric is compared with that of the traditional Shannon-Hartley theorem-based metric.

2 System-level performance evaluation of MU-MIMO THP

2.1 Operating principle of MU-MIMO THP

In this section, we briefly introduce the operating principle of MU-MIMO THP with user scheduling. Fig. 1 shows the system configuration, where $N_t$ and $N_r$ denote the number of base station (BS) antennas and MSs with one received antenna element, respectively. In Fig. 1, user scheduling is performed prior to MU-MIMO THP to select the suitable MSs and then, the feedforward (FF) and feedback (FB) filters are determined to retain spatial orthogonality among selected MSs.

In general, THP can be implemented by an LQ decomposition [6, 7], and the channel matrix $H \in \mathbb{C}^{N_r \times N_t}$ can be decomposed as

$$H = LQ,$$

where $L \in \mathbb{C}^{N_r \times N_r}$ and $Q \in \mathbb{C}^{N_r \times N_t}$ are lower triangular and unitary matrices, respectively. Assuming that the precoding weight is determined by the zero-forcing (ZF) criterion, both FF filter $F$ and FB filter $B$ for the THP algorithm can be obtained as

$$G = \text{diag}\{L_{11}^{-1}, \cdots, L_{N_r,N_r}^{-1}\},$$

$$F = Q^H G,$$

$$B = HF - I = LG - I,$$

where $L_{ii}$ is the $i$-th diagonal element of $L$.

In THP, the modulo operation is performed to limit the transmit power increased by the addition of an interference subtraction vector generated by the FB filter $B$. 

$$\text{(1)}$$

$$\text{(2)}$$

$$\text{(3)}$$

$$\text{(4)}$$
Moreover, because the transmit power is changed by the FF filter $F$, a power normalization factor $g$ is required, which is given by

$$g = \sqrt{\frac{\text{tr}(FC_vF^H)}{E_{tx}}},$$

(5)

where $E_{tx}$ denotes the total transmit power and $C_v \in \mathbb{C}^{N_r \times N_r}$ is the covariance matrix of the transmit signal after the modulo operation $v \in \mathbb{C}^{N_r}$.

### 2.2 Mod-Λ channel-based analysis for MU-MIMO THP

The system capacity of MU-MIMO can be generally analyzed using the power normalization factor $g$ shown in Eq. (5). This is because this normalization factor indicates the SNR. Therefore, the sum-rate based on the Shannon-Hartley theorem is given by [6, 7]

$$C_{\text{sum}} = \sum_{i=1}^{N} \log_2 \left( 1 + \frac{\sigma^2 \tau}{g^2 \sigma_n^2} \right) \text{[bps/Hz]},$$

(6)

where $\sigma^2$ and $\sigma_n^2$ are the transmit signal power and noise power, respectively.

The approach shown in Eq. (6) has the problem that the impact of the modulo loss resulting from the THP modulo operation at the receiver is not taken into account. Therefore, in this paper, we investigate the system-level performance of THP in consideration of the modulo loss. In detail, the impact of the modulo loss is considered using the mod-Λ channel-based analysis [12], and it is clarified by comparing it with the traditional Shannon-Hartley-based approach.

The achievable rate of the mod-Λ channel is given by

$$C = 2(\log_2 \tau - H(Z_{\text{mod}})) \text{ [bps/Hz]},$$

(7)

where $\tau$ and $H(Z_{\text{mod}})$ denote the modulo width and differential entropy of the white Gaussian noise (WGN) after the modulo operation. Thus, in order to obtain the differential entropy $H(Z_{\text{mod}})$, it is necessary to derive the probability density function (PDF) of the WGN after the modulo operation $p_{Z_{\text{mod}}}(z_{\text{mod}})$. The PDF of the WGN $p_Z(z)$ is represented by

$$p_Z(z) = \frac{1}{\sqrt{2\pi g^2 \sigma_n^2}} e^{-\frac{z^2}{2g^2\sigma_n^2}}.$$

(8)

The actual impact of the WGN after the modulo operation is represented as the sum of shifted versions of the PDF $p_Z(z)$ in the fundamental region $[-\tau/2, \tau/2]$. The shifts are integral multiples of the modulo width $\tau$. Thus, the PDF of the WGN after the modulo operation $p_{Z_{\text{mod}}}(z_{\text{mod}}) (-\tau/2 < z_{\text{mod}} < \tau/2)$ is given by
In consequence, the sum-rate of MU-MIMO THP is represented as

\[ C_{\text{sum}} = \sum_{i=1}^{N_t} 2 \left( \log_2 \tau - H(Z_{\text{mod}}) \right) \]

\[ = \sum_{i=1}^{N_t} 2 \left( \log_2 \tau + \int_{-\tau/2}^{\tau/2} p_{Z_{\text{mod}}}(z_{\text{mod}}) \log_2 p_{Z_{\text{mod}}}(z_{\text{mod}}) dz_{\text{mod}} \right) \text{ [bps/Hz].} \] (10)

2.3 Application of user scheduling to MU-MIMO THP

User scheduling is generally performed before precoding because the number of MSs in a radio zone is more than the number of BS antennas. In this paper, we consider PF [8] as user scheduling and analyze the system-level performance using the above-mentioned mod-Λ channel-based approach.

In application of PF to MU-MIMO THP, the system capacity of all possible combinations of MSs has to be calculated because it is used as a criterion in PF-based user selection. The PF metric in the \( k \)-th combination \( M_k \) is given by

\[ M_k = \sum_{i=1}^{N_t} \frac{R_{k,i}(t)}{T_{k,i}(t)} \quad (k = 1, 2, \cdots, K_{\text{IC}}), \] (11)

where \( K \) is the number of the existing users, and \( R_{k,i}(t) \) is the instantaneous system capacity of the \( i \)-th MS, in the case that the \( k \)-th combination is admitted to the transmission at time \( t \). \( T_{k,i}(t) \) is average system capacity of the \( i \)-th MS in the \( k \)-th combination until time \( t \), which is represented as

\[ T_{k,i}(t + 1) = \left( 1 - \frac{1}{t_c} \right) T_{k,i}(t) + \frac{1}{t_c} R'_{k,i}(t), \] (12)

where \( t_c \) is the average time range of the system capacity and \( R'_{k,i}(t) \) is the instantaneous system capacity of the \( i \)-th MS at time \( t \). Here, \( R'_{k,i}(t) \) is zero if the \( i \)-th MS is not scheduled at time \( t \). Eqs. (11) and (12) proves that the PF metric requires the instantaneous system capacity, which is obtained from Eq. (6) or (10).

3 Numerical results

In this section, we evaluate system-level performance of MU-MIMO THP with PF user scheduling based on the mod-Λ channel-based approach, and compare its performance to that of the traditional Shannon-Hartley theorem-based approach with or without considering modulo loss. Fig. 2 shows the evaluation model and its system parameters. In our performance evaluation, MSs are randomly distributed and the ordering process [7, 12] is adopted to enhance the transmission performance of THP. In addition, the MIMO channel is assumed to follow spatially uncorrelated Rayleigh fading. Moreover, perfect channel state information (CSI) feedback is assumed, and its feedback error and delay are negligible.

Fig. 3 shows the sum-rate versus the number of existing users \( K \), where the MIMO antenna configurations are set to be \( 4 \times 4 \) and \( 6 \times 6 \). Fig. 3(a) demonstrates that the performance of the mod-Λ channel-based analysis is lower than that of the traditional Shannon-Hartley theorem-based approach, regardless of MIMO antenna
configuration, which indicates that the traditional approach overestimates its performance. To clarify the required accuracy of the PF metric, in Fig. 3(b), the sum-rate is obtained from the mod-Λ channel-based and traditional Shannon-Hartley theorem-based PF metrics. From Fig. 3(b), the sum-rates of both approaches are the same regardless of the MIMO antenna configuration, which implies that the traditional Shannon-Hartley theorem-based approach is only useful for PF metric calculation.

4 Conclusion

In this paper, we presented the exact system-level performance of MU-MIMO THP with PF user scheduling by means of the mod-Λ channel-based analysis. Moreover, we clarified the required accuracy of PF metric by comparing the performance of the mod-Λ channel-based PF metric and the traditional Shannon-Hartley theorem-based metric. Numerical results showed that the system-level performance of mod-Λ channel-based analysis is slightly lower than that of the traditional Shannon-Hartley theorem-based approach, which implies that the traditional approach overestimates its performance. However, in use of the PF metric, the traditional Shannon-Hartley theorem-based approach can be adopted because there is no performance difference between these two approaches.