Spectrum sensing based on weighted diversity combining using time-averaged CAF†

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Abstract: This paper proposes a cyclic autocorrelation function (CAF) diversity combining technique for spectrum sensing using test statistics shared among multiple receive antennas with time-averaged weights. The proposed technique computes a weight factor by averaging CAF, and the weight factor is employed to improve the performance of signal detection. The proposed results are compared with some conventional techniques, and they show that the signal detection performance can be improved without the increasing computational complexities in comparison with the conventional techniques.

Keywords: cognitive radio networks, spectrum sensing, cyclostationary, space diversity, OFDM signal

Classification: Terrestrial Wireless Communication/Broadcasting Technologies

References


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1 Introduction

Cognitive radio is the core technology employed for efficient use of high frequency bands. In cognitive radio networks, secondary users need to sense and monitor the radio spectrum to detect and share the frequency bands that are not occupied by primary users (PUs). Therefore, spectrum sensing techniques are very important for realizing cognitive radio networks [1]. Various traditional spectrum sensing techniques based on cyclostationary detection, which does not need to measure a noise floor [2, 3], have been proposed. We have reported a spectrum sensing technique based on maximum cyclic autocorrelation selection (MCAS) [3] using multiple receive antennas [4, 5] with a low computational complexity. However, the technique [4] uses the weight factor computed from an instantaneous value of a cyclic autocorrelation function (CAF) and the noise component included in the weight factor negatively affects the performance. Further, the technique [5] is based on equal gain combining technique, and a weighting technique without increasing the complexity for the computation of the weight factor is expected. This paper proposes a CAF diversity combining technique for spectrum sensing using test statistics shared among multiple receive antennas with time-averaged weights as an extension of [5].

2 Preliminary notion

2.1 Spectrum sensing

This paper considers spectrum sensing of orthogonal frequency division multiplexing (OFDM) signals with the number of samples for a useful symbol duration \( N_{\text{FFT}} \) and cyclic prefix (CP) duration \( N_{\text{CP}} \), and \( N_{\text{OFDM}} = N_{\text{FFT}} + N_{\text{CP}} \), where \( N_{\text{OFDM}} \) is the number of samples for the OFDM symbol. The spectrum sensing problem in cognitive radio networks can be considered as a binary hypothesis testing problem. We let \( H_1 \) and \( H_0 \) denote the hypothesis in which the PU is active and inactive, respectively, and these can be written as

\[
H_1 : r_i(n) = x_i(n) + v_i(n), \quad i = 1, \ldots, N_R, \\
H_0 : r_i(n) = v_i(n)
\]

(1)

where \( r_i(n) \), \( x_i(n) \), \( v_i(n) \) and \( N_R \) are the received signal, received PU signal, additive white Gaussian noise with a mean of zero and variance \( \sigma_v^2 \) and the number of receive antenna, respectively, and the subscript \( (\cdot)_i \) represents the receive antenna. Also, \( x_i(n) = h_i s(n) \) where \( s(n) \) and \( h_i \) are PU’s OFDM signal and channel gain, respectively.
2.2 Test statistics sharing among multiple receive antennas based on MCAS

In MCAS, the CAF values at cyclic frequencies, which are \( a_1(a_1 = 1/N_{OFDM}) \) and \( \beta_k (\beta_k = (k + 0.5)/N_{OFDM}, k = 0, \ldots, N_D - 1) \), are used to sense OFDM signals where \( N_D \) is the number of CAFs at \( \beta_k \). Note that \( N_D \) determines a target false alarm probability \( P_{FA} \) for signal detection, and \( P_{FA} = 1/(N_D + 1) \). The CAFs at these frequencies are given by

\[
\hat{R}_{\gamma_k}^{a_i}(v) = \frac{1}{N} \sum_{n=0}^{N-1} r_i(n)r_i^*(n + v)e^{-j2\pi \gamma_k n \Delta t}, \quad i = 1, \ldots, N_R, \tag{2}
\]

\[
\hat{R}_{\gamma_k}^{b_i}(v) = \frac{1}{N'} \sum_{n=kN'}^{(k+1)N'-1} r_i(n)r_i^*(n + v)e^{-j2\pi \gamma_k n \Delta t}, \quad i = 1, \ldots, N_R, k = 0, 1, \ldots, N_D - 1, \tag{3}
\]

where \((\cdot)^*\), \(v\), \(\Delta t\), \(N\) and \(N'\) are the complex conjugate, a delay, the sampling interval, the number of samples used to compute \( \hat{R}_{\gamma_k}^{a_i}(v) \) and \( \hat{R}_{\gamma_k}^{b_i}(v) \), respectively, and \( N_D N' \leq N \). Note that \( \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) \) corresponds to a peak of CAF and \( \hat{R}_{\gamma_k}^{b_i}(N_{FFT}) \)s are not peak of CAF in \( \mathcal{H}_1 \), and \( \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) \) and \( \hat{R}_{\gamma_k}^{b_i}(N_{FFT}) \) follow \(\mathcal{CN}(0, \sigma_k^2/N)\) and \(\mathcal{CN}(0, \sigma_k^2/N')\) in \( \mathcal{H}_0 \), respectively. In the conventional technique [5], the CAFs at \( \beta_k \) are computed from not all received signals, but only one received signal, and the statistics for signal detection in terms of two cyclic frequencies are given by \( \overline{T}_{\alpha_i} = |\frac{1}{N_R} \sum_{i=1}^{N_R} \hat{R}_{\gamma_k}^{a_i}(N_{FFT})| \) and \( \overline{T}_{b_i} = |\hat{R}_{\gamma_k}^{a_i}(N_{FFT})| \), \( \forall i = 1, \ldots, N_R \). The spectrum sensing can be carried out as

\[
\overline{T}_{\alpha_i} \geq \max_k \sqrt{\frac{N'N}{NN_R}} \overline{T}_{b_i}, \quad k = 0, 1, \ldots, N_D - 1. \tag{4}
\]

3 CAF diversity combining by test statistics sharing with time-averaged weights

In this paper, we propose a weighted statistic for signal detection in terms of \( a_1 \) to improve the performance of the spectrum sensing, and it can be written as

\[
\overline{T}_{\alpha_i} = \sum_{i=1}^{N_R} w_i^{a_i} \hat{R}_{\gamma_k}^{a_i}(N_{FFT}), \tag{5}
\]

where \( w_i^{a_i} \) is the weight factor for \( \hat{R}_{\gamma_k}^{a_i}(v) \). In [6], it was shown that the signal detection performance can be improved by using a weight factor \( g_{opt,i} \), which is given by \( g_{opt,i} = |h_i|^2/\sqrt{\sum_{j=1}^{N_R} |h_j|^4} \), \( i = 1, \ldots, N_R \). Although \( w_i^{a_i} \) can be derived from the channel gain as the general maximum ratio combining, it is difficult to estimate the channel gain because the received signal strength is very small in the case of spectrum sensing. In order to obtain the weight factor \( w_i^{a_i} \), we utilize \( \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) \) which includes an information of the channel gain. Because \( \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) \) is the approximated CAF value and includes a noise component, we first consider \( \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) \) for \( N \to \infty \) as

\[
\lim_{N \to \infty} \hat{R}_{\gamma_k}^{a_i}(N_{FFT}) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} [x_i(n)x_i^*(n + N_{FFT}) + x_i(n)v_i^*(n + N_{FFT}) + v_i(n)x_i^*(n + N_{FFT}) + v_i(n)v_i^*(n + N_{FFT})] e^{-j2\pi \gamma_k n \Delta t}. \tag{6}
\]
Assuming all the terms except the first are 0 when $N \to \infty$,

$$\lim_{N \to \infty} R_i^\alpha(N_{\text{FFT}}) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} |h_j|^2 s(n)s^\ast(n + N_{\text{FFT}}) e^{-j2\pi n t} \approx |h_j|^2 R_i^\alpha(N_{\text{FFT}}),$$

(7)

where $R_i^\alpha(N_{\text{FFT}})$ is a CAF of $s(n)$ at cyclic frequency $\alpha_1$ when $N \to \infty$. In this manner, the noise component is completely removed, and the channel gain can be obtained. However, $N \to \infty$ is impossible, and we attempt to remove the noise component in $\hat{R}_i^\alpha(N_{\text{FFT}})$ by averaging $\hat{R}_i^\alpha(N_{\text{FFT}})$. The proposed technique successively averages the $\hat{R}_i^\alpha(N_{\text{FFT}})$s computed at the $N_T$ times of sensing operations.

The time-average of the $\hat{R}_i^\alpha(N_{\text{FFT}})$s, which are used for sensing at the time $tN$, is given by

$$\bar{R}_i^\alpha(tN) = \frac{1}{N_T} \sum_{m=-N_T+1}^{t} \hat{R}_i^\alpha(N_{\text{FFT}}, mN),$$

(8)

where $\hat{R}_i^\alpha(v, mN)$ is the newly defined $\hat{R}_i^\alpha(v)$ at the time $mN$. Similarly, we let $w_i^\alpha(tN)$ denote the newly defined $w_i^\alpha(tN)$ at the time $tN$, and $w_i^\alpha(tN)$ is defined as

$$w_i^\alpha(tN) = \frac{|R_i^\alpha((t-1)N)|}{\sqrt{\sum_{j=1}^{N_R} |R_j^\alpha((t-1)N)|^2}}.$$  

(9)

Because $R_i^\alpha((t-1)N) \approx |h_j|^2 R_i^\alpha(N_{\text{FFT}})$ when $N_T$ is a sufficiently large, $w_i^\alpha(tN)$ can be approximated as

$$w_i^\alpha(tN) \approx \frac{|R_i^\alpha(N_{\text{FFT}})|^2}{\sqrt{\sum_{j=1}^{N_R} |R_j^\alpha(N_{\text{FFT}})|^2}} = \frac{|h_j|^2}{\sqrt{\sum_{j=1}^{N_R} |h_j|^2}} = g_{\text{opt},j}.$$  

(10)

Further, we let $T_{\alpha, i}(tN) = |\sum_{j=1}^{N_R} w_i^\alpha(tN) \hat{R}_j^\alpha(N_{\text{FFT}}, tN)|$ and $T_{\beta, i}(tN) = |\hat{R}_i^\alpha(N_{\text{FFT}}, tN)|$, $\forall (i = 1, \ldots, N_R)$ denote newly defined $T_{\alpha, i}$ and $T_{\beta, i}$ at the time $tN$, respectively. Note that $R_i^\alpha((t-1)N)$ is used for the computation of $w_i^\alpha(tN)$ so as not to change the statistical properties of $T_{\alpha, i}(tN)$, i.e., it is to avoid that $\hat{R}_i^\alpha(N_{\text{FFT}}, tN)$ is included in $w_i^\alpha(tN)$ which multiplies with $\hat{R}_i^\alpha(N_{\text{FFT}}, tN)$. From these, the decision criterion of the proposed technique is given by

$$T_{\alpha, i}(tN) \geq \max_k \sqrt{\frac{N}{N}} T_{\beta, k}(tN), \quad k = 0, 1, \ldots, N_D - 1.$$  

(11)

Finally, we discuss the computational complexity of the proposed technique. We consider the complexity required for multiplication of complex variables. The proposed technique requires the computation of CAFs at $\alpha_1$ and $\beta_k$ using signal detection and the weighting of peak CAFs. In addition, we assume that $N_T$ does not affect the computational complexity because the CAFs computed at the past sensing time are considered to be diverted for computation of the weight factor. Thus, the complexity of the proposed technique can be written as $C_{\text{w-share}} = 8N_R + 4N'N_D + 2(N_D + 1) + 4N_R$. Next, the computational complexities of the conventional multiple receive antenna based techniques [2, 4, 5] are compared with that of the proposed technique. The computational complexities of the conventional techniques are given by $C_{\text{weight}} = 2(4N + 2(N' + 1)N_D + 1)N_R$, $C_{\text{share}} = 8NN'_R + 4N'N_D + 2(N_D + 1)$ and $C_{\text{Huang}} = 8NN'_R + 2(N + 2)N_R$ where $C_{\text{Huang}}$, $C_{\text{weight}}$, and $C_{\text{share}}$ are the computational complexity of the techniques proposed in [2], [4], and
Assuming $N_D = N$ and $N$ is larger than the numerators of the terms, the ratios of the complexities of the proposed technique and conventional techniques can be written as

$$\frac{C_{\text{w-share}}}{C_{\text{Huang}}} = \frac{8 + 4/N_R + 2(N_D + 1)/NN_R + 4/N}{8N_R + 2 + 4/N} \approx \frac{8 + 4/N_R}{8N_R + 2},$$  \hspace{1cm} (12)

$$\frac{C_{\text{w-share}}}{C_{\text{weight}}} = \frac{8 + 4/N_R + 2(N_D + 1)/NN_R + 4/N}{12 + 4N_D/N + 2/N} \approx \frac{8 + 4/N_R}{12},$$  \hspace{1cm} (13)

$$\frac{C_{\text{w-share}}}{C_{\text{share}}} = \frac{8 + 4/N_R + 2(N_D + 1)/NN_R + 4/N}{8 + 4/N_R + 2(N_D + 1)/NN_R} \approx \frac{8 + 4/N_R}{8 + 4/N_R}. \hspace{1cm} (14)$$

Fig. 1 shows $C_{\text{w-share}}/C_{\text{Huang}}$, $C_{\text{w-share}}/C_{\text{weight}}$ and $C_{\text{w-share}}/C_{\text{share}}$ for some $N_R$s. It can be seen that the ratio of the computational complexities of the proposed and conventional technique [5] is 1 regardless of $N_R$. Further, the ratios of computational complexities of the proposed and conventional techniques [2, 4] decrease as $N_R$ increases.

### 4 Numerical examples

To evaluate the performance of the proposed technique, numerical examples are shown in this section. The PU signal is OFDM with $N_{\text{FFT}} = 64$ and $N_{\text{CP}} = 16$. A Rayleigh flat fading channel model is employed. Further, we assume that $\overline{P}_{\overline{T}A} = 0.1$ ($N_D = 9$), $N = 2560$ and $N' = 284$. Fig. 2 shows the performance of signal detection probability for the proposed technique and conventional techniques. In Fig. 2, the proposed technique ($N_T = 25$), the proposed technique with optimum weight ($g_{\text{opt,t}}$), and the three conventional techniques [2, 4, 5] are compared. It can be seen that the performance of the proposed technique is almost the same as that of the technique using $g_{\text{opt,t}}$. Furthermore, the proposed technique can demonstrate better performance than the conventional techniques. In particular, the performance
of the proposed technique is almost the same as that of the technique [2] although the computational complexity of the proposed technique is less than that of the technique.

5 Conclusion

This paper proposed a CAF diversity combining technique for spectrum sensing using test statistics shared among multiple receive antennas with time-averaged weights. The proposed technique can reduce the noise component in the weight factor by averaging CAF containing information of the channel gain, and we attempt to improve the signal detection performance by using the weight factor. Numerical examples showed that the proposed technique can improve the performance of signal detection compared with that of conventional techniques without increasing the computational complexity.

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ALOS-2 polarimetric SAR observation of Hokkaido-Iburi-Tobu earthquake 2018

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Abstract: This paper reports prompt space-borne ALOS-2 SAR observation results of Hokkaido-Iburi-Tobu earthquake on 2018/09/06. Emphasis is placed on quick survey for disaster monitoring using fully polarimetric data. On 2018/09/08, ALOS-2 has acquired data over the disaster area. By comparison of the previous data (2017/08/26) before the earthquake, damaged areas by landslides are clearly detected. The analysis is based on the scattering power decomposition, which retrieves scattering mechanism change from bare soil surface caused by landslide and serves to identify the landslide location. The decomposition and anisotropy images are presented to show the effectiveness of fully polarimetric SAR sensing from space.

Keywords: polarimetric synthetic aperture radar, scattering power decomposition, disaster monitoring, landslide detection

Classification: Sensing

References

1 Introduction

Hokkaido-Iburi-Tobu earthquake occurred on 2018/09/06 with a magnitude of 6.7 and seismic intensity of 7 [1]. This earthquake destroyed everything for daily life and caused devastating damages, including tremendous landslides of 13.4 km² around Atsuma-cho area. Advanced Land Observing Satellite 2 (ALOS-2) was observing the situation on 2018/09/08. ALOS-2/PALSAR-2 is Japanese space-borne synthetic aperture radar (SAR) at L-band developed by JAXA, and has been operating for disaster monitoring [2, 3]. Among various SAR systems and their applications [4], L-band SAR has advantages of penetration of clouds and also penetration into forests, leading to excellent operating capability in all weather condition at anytime for monitoring the earth-cover. Since the data acquisition mode was fully polarimetric, it became possible to retrieve scattering mechanism from the imaging scene, to compare the previous data set on 2017/08/26, and to investigate the difference in this area. Fig. 1 shows the earthquake location and ALOS-2 observation area. This paper reports the prompt imaging results due to the earthquake event.

2 Quad Pol. Data by ALOS-2

The acquired data numbers by ALOS-2 are ALOS2176020840-170826 and ALOS2231910840-180908. The specifications of the data are: off-nadir angle = 28°, Center latitude = 42.34°, Center Longitude = 142.2°, 8950 × 22187 pixels with range pixel size ~6 m and azimuth pixel size ~3 m, in the ascending orbit with right direction looking. The data covers approximately 50 km (range) by 70 km (azimuth) as shown in Fig. 1(a). Fig. 1(b) is one of the landslide areas, which has frequently appeared on TV news.

3 Decomposition images

We used the time series data sets for polarimetric imaging and change detection based on the four-component scattering power decomposition [5]. The scattering
power decomposition yields surface scattering power $Ps$, double bounce scattering power $Pd$, volume scattering power $Pv$, and helix scattering power $Ph$. These powers are color-coded to create a full color image which is comprehensive to everybody.

Fig. 2 shows decomposition image over Atsuma-cho area, Hokkaido, using a special RGB color-coding [6] with Red for $Ps$, Green for $Pv$, and Blue for $Pd$. The reason of using this color-coding is to enhance the contrast and to attract attention to the bare soil surface caused by landslide induced by the earthquake. This color-coding enhances the surface scattering by bare soil surface distinctively from other objects and serves to identify the landslide location. Fig. 2 is a movie file comparing with “before” and “after” the earthquake. It is possible to find the changes by color, i.e., by our eye inspection. It is seen in Fig. 2 that Red areas are spread all over the scene due to the earthquake.

Red color areas in Fig. 2 are characterized by the surface scattering power $Ps$ dominant due to bare soil surface scattering. The $Ps$ has increased significantly (up to 6 dB) in the landslide areas. Since bare soil surface reflects the surface scattering power in omni-direction, it is rather easily picked up by SAR and recognized by color in the decomposition image.

In addition, small mountain ridge moved to the left direction by this earthquake. We can recognize the ridge movement (indicated by a small pink arrow) at the lower left of the 6-th and 7-th images of Fig. 2. Movie image has advantage of showing the movement.

![Fig. 2. Scattering power decomposition images before and after the earthquake of Atsuma-cho, Hokkaido. RGB color-coding with R($Ps$), G($Pv$) and B($Pd$).](image-url)
4 Change detection

As can be seen in Fig. 2, it is possible to identify the location where color (scattering mechanism) has changed. These changed areas are caused by the earthquake. Some areas have increasing $P_s$, whereas other areas have decreasing tendency. At the same time, there are also areas with increasing or decreasing $P_v$. In this section, we try to pick up these areas as much as possible using some polarimetric information.

Fully polarimetric data provides the total power $TP$, which is the most fundamental radar quantity, accounting for all polarimetric combination (HH, HV, VV) powers.

$$TP = |S_{HH}|^2 + 2|S_{HV}|^2 + |S_{VV}|^2.$$ (1)

where $S_{HH}$, $S_{HV}$, and $S_{VV}$ are elements of scattering matrix $[S]$. This value is invariant with respect to change of polarization basis. In the four-component scattering power decomposition [5], it can be also represented as a sum of the surface scattering power $P_s$, the volume scattering power $P_v$, the double bounce scattering power $P_d$, and the helix scattering power $P_h$.

$$TP = P_s + P_d + P_v + P_h.$$ (2)

We can define the following normalized “Anisotropy parameter A” for change detection,

$$A(TP) = \frac{TP^a - TP^b}{TP^a + TP^b},$$ (3)

where the superscript “a” shows “after” the event while “b” means “before” the event. This value falls in the range $[-1, 1]$ and indicates how the total power is changed. This normalized value can be a good indicator for monitoring of disaster area.

In a similar way, we can define the anisotropy parameter $A$ for each scattering power,

$$A(P_s) = \frac{P_s^a - P_s^b}{P_s^a + P_s^b}, \quad A(P_d) = \frac{P_d^a - P_d^b}{P_d^a + P_d^b},$$

$$A(P_v) = \frac{P_v^a - P_v^b}{P_v^a + P_v^b}, \quad A(P_h) = \frac{P_h^a - P_h^b}{P_h^a + P_h^b}.$$ (4)

$$A(S_{HH}) = \frac{|S_{HH}^a|^2 - |S_{HH}^b|^2}{|S_{HH}^a|^2 + |S_{HH}^b|^2}, \quad A(S_{HV}) = \frac{|S_{HV}^a|^2 - |S_{HV}^b|^2}{|S_{HV}^a|^2 + |S_{HV}^b|^2},$$

$$A(S_{VV}) = \frac{|S_{VV}^a|^2 - |S_{VV}^b|^2}{|S_{VV}^a|^2 + |S_{VV}^b|^2}. $$ (5)

By this notation, it is possible to examine the change of scattering mechanisms or of polarimetric channel power. Since these values are normalized in $[-1, 1]$, a simple addition of these parameters may cancel out in some cases. Rather, the magnitude sum of these parameters would yield the change profoundly. Therefore the following parameter may be suitable for the change detection purpose.

$$TA = |A(P_s)| + |A(P_d)| + |A(P_v)| + |A(P_h)|$$ (6)
Having defined these parameters (2)–(6) and through many trials to the ALOS-2 data sets, we came to the final criteria that damaged area was well retrieved by $|A(\text{TP})| > 0.35$ and $|A(P_v)| > 0.4$ among others.

Fig. 3 shows the change detection result. Upper image of Fig. 3 is derived by the volume scattering power anisotropy $A(P_v)$. The reason why we have chosen $A(P_v)$ is such that the volume scattering power is mainly generated by trees in this area, and is lost if landslide occurs. If landslide happens, land-cover changes from trees to bare soil. In this case $P_v$ by trees decreases. Considering this scattering mechanism change, we have chosen the criteria for decreasing case as $A(P_v) < -0.4$ after some trials. The corresponding area is colored with Pink. On
the other hand, $P_v$ increases if trees or vegetation come into the corresponding area. These scattering mechanism changes could be a good indicator for landslide. The criteria for increasing case was chosen as $A(P_v) > 0.4$, and the area is colored with Cyan. By using a median filter with $5 \times 5$ window, change detection image is displayed in Fig. 3.

Pink color areas are mostly corresponding to the landslide locations in the upper image of Fig. 3. Using a reference by aerial-photo [1], we can confirm the areas correspond to actual situation. Fig. 1(b) photo area is clearly detected in a small white rectangle in Fig. 3.

Lower image of Fig. 3 can be used for quick monitoring of earthquake using $TP$ magnitude information. If $|A(TP)|$ is employed, any changes can be picked up by the mono-color detection result. Mono-color would be suitable for quick survey. On the other hand, $A(P_v)$ image can be used for the change detection in more detail exhibiting specific scattering mechanism change, i.e., increasing or decreasing. This feature can be recognized, for example, in a small white rectangular area. Although there are lots of polarimetric parameters such as (4), (5) or each decomposition power itself, the volume scattering power $P_v$ and $A(P_v)$ and $A(TP)$ have played the most important role for landslide detection in this earthquake. The key parameter might change according to disaster type, e.g., it was $Ps$ for Tsunami disaster [3] of the great East Japan Earthquake on 2011/03/11.

5 Conclusion

Japanese space-borne sensor “ALOS-2” SAR has acquired fully polarimetric data over Hokkaido-Iburi-Tobu region, where a great earthquake occurred on 2018/09/06 with magnitude of 6.7. Emphasis is placed on quick survey for disaster monitoring using fully polarimetric data. By comparison of the previous data (2017/08/26) before the earthquake, damaged areas by landslides are clearly detected. The scattering power decomposition and power anisotropy method were successfully applied to detect damaged areas. Time series images before and after the earthquake as well as change detection image were presented to show the effectiveness of fully polarimetric SAR sensing from space. It is really important to monitor the earth by fully polarimetric mode routinely.

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Observed results on optimal sample support in space time adaptive processing in radar system

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Abstract: Space Time Adaptive processing (STAP) has been proposed to suppress the clutter in the airborne radar. In STAP covariance matrix of the received signal is employed to carry out the processing. This is often estimated by making use of returns from neighboring range bins of interest. In many simulation results, independent and identically distributed (IID) samples are assumed for the clutter. It is said that the more samples we consider in sample covariance matrix, the better signal-to-interference plus noise ratio (SINR) is achieved in STAP. However, we have found the optimum sample size to achieve maximum SINR in real circumstance. In this paper, we discuss the violation of IID assumption resulting in degradation of SINR using real sea clutter data observed by S-band side-looking airborne radar. The existence of the optimum samples is shown here.

Keywords: radar, clutter, signal processing, STAP

Classification: Sensing

References

1 Introduction

Space Time Adaptive Processing (STAP) is proposed to suppress the clutter in airborne radars [1, 2, 3, 4]. In STAP the covariance matrix of the received signal is employed. The covariance matrix $\mathbf{R}$ and the optimum weight $\mathbf{w}$ are (1) and (2) respectively.

$$\mathbf{R} = \mathbb{E}\{\mathbf{C} \cdot \mathbf{C}^H\}$$

(1)

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{a}$$

(2)

where $\mathbf{C}$ and $\mathbf{a}$ denote the space-time interference vector corresponding to the clutter and uncorrelated receiver noise and the space-time steering vector corresponding to the target direction respectively. $\mathbb{E}$ is the operator to estimate its expected value, $\mathbb{H}$ denotes Hermitian transposition.

In real situation, the sample covariance matrix is used and the received signal is used instead of $\mathbf{C}$ with avoiding self-nulling.

The sample covariance matrix $\hat{\mathbf{R}}$ is estimated by making use of returns from neighboring range bins of interest described in (3).

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{X}_k \cdot \mathbf{X}_k^H$$

(3)

where $\mathbf{X}_k$ denotes the space-time received sample vector for $k$th range bin and $K$ denotes the number of samples used for processing.

Therefore the corresponding weight vector $\hat{\mathbf{w}}$ is described in (4).

$$\hat{\mathbf{w}} = \hat{\mathbf{R}}^{-1} \mathbf{a}$$

(4)

The processed SINR depends on the number of the samples. In [5], the more samples are employed, the better SINR can be achieved. In [5], the ratio of the expected SINR to optimum SINR $\rho$ is described in (5).

$$\rho = \frac{K + 2 - N}{K + 1}, \quad K \geq N$$

(5)
where $N$ is the degree of freedom, which is the product between the number of antenna elements and the number of the processed pulses during coherent processing interval.

According to previous studies [6], $K$ should be greater than or at least equal to $N$ because $\mathbf{R}$ must be nonsingular. When $N$ is a large number, we should sample the data up to the far range bin from the target range bin. Since the clutter property of far range bins is different from that of the target range bin, the accuracy of the estimated $\hat{\mathbf{R}}$ may be degraded.

The simulation results [6] assume that all sample range bins are satisfied with IID, however the clutter property may not always satisfy IID in real environment. That indicates the IID assumption is violated in some real circumstances. Therefore employing too many samples for estimating $\hat{\mathbf{R}}$ may degrade the processed SINR after STAP.

To evaluate the effects of the violation and to show the optimum number of samples, we have analyzed the real sea clutter data observed in S-band at the Sea of Japan in winter by the side-looking radar. In case of less samples used for the sample covariance matrix, $\hat{\mathbf{R}}^{-1}$ may be singular or inaccurate matrix. Therefore we use the diagonal loading [7] for the small sample case.

Some observed results of STAP are shown in [8, 9]. In their studies, they are measured in X-band and HF and the observed clutter are ground clutter and ionospheric clutter respectively, which are different from sea clutter. And, they did not discuss the relation between the number of samples to make sample covariance matrix $\hat{\mathbf{R}}$ and its resultant SINR.

In this paper, we introduce the STAP performance data about real sea clutter, which have seldom been reported. Furthermore, we discuss the violation of IID assumption resulting in degradation of SINR using real sea clutter data observed in S-band. The existence of the optimum samples is shown here.

2 Method of processing

We explain the method of signal processing to make sure of the effect on STAP.

First, we explain the way of making sample covariance matrix $\hat{\mathbf{R}}$ (3). The received samples are taken from range bins around the target. $K/2$ samples of near ranges and $K/2$ samples of far ranges are used to estimate $\hat{\mathbf{R}}$, the center of which is the target range bin. Note that 2 guard cells for both side are not used as samples to avoid self-nulling [2]. Thus, we estimated $\hat{\mathbf{R}}$ various samples up to 1024.

Though fully adaptive STAP is difficult to compute $\hat{\mathbf{R}}^{-1}$ due to the large matrix size, it is employed in this paper. That is why we want to evaluate the clutter property, though some algorithms are proposed such as JDL STAP [10] in order to reduce the computational load.

Next, we show the way of calculation of SINR. $S$ (Signal) is the amplitude of target’s range bin. I plus N (interference plus noise) are the average of amplitude neighboring 128 range bins except 2 guard cells. Then, we calculate $S/(I+N)$ and express it in decibel.

Note that ordinary S/N optimization filter method is to use only a steering vector as a weight vector,
where \( \mathbf{a} \) denotes the steering vector which is same as weight of STAP (2). After summing the received signals with (6), we applied the pulse compression.

3 Measurement of the clutter

We have observed the sea clutter at the Sea of Japan in winter by the side-looking radar. The radar is mounted on the left side of the aircraft. The array is a uniform linear array which consists of 8 elements. The measurement parameters are shown in Table I. According to the Table I, \( N \) is 256 which corresponds to the product between number of antenna elements 8 and number of processed pulses 32.

The sea state during the measurement is estimated to be 1 according to the wind speed which was 2.0 m/s on average.

### Table I. The measurement parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Setting Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft speed</td>
<td>91.47 m/s (177.8 knots)</td>
</tr>
<tr>
<td>Aircraft altitude</td>
<td>6,102 m (20,019 feet)</td>
</tr>
<tr>
<td>Frequency band</td>
<td>S-band</td>
</tr>
<tr>
<td>Pulse Repetition Frequency</td>
<td>218 Hz</td>
</tr>
<tr>
<td>Array antenna’s interval</td>
<td>0.47 m</td>
</tr>
<tr>
<td>Number of the processed pulses</td>
<td>32</td>
</tr>
<tr>
<td>Number of antenna elements</td>
<td>8</td>
</tr>
<tr>
<td>Range bin</td>
<td>30 m</td>
</tr>
</tbody>
</table>

4 Results

In this section, we show the results of STAP which is applied for real sea clutter data received in S-band. We also show the comparison between SINR after STAP and one after the ordinary method.

4.1 Comparison between STAP and ordinary method

Fig. 1 shows the A-scope of the sea clutter and the real target signal. The target is a real ship. Amplitude is normalized with the maximum signal (or target’s signal). At the 1,271st range bin, around 60 km, the target is detected and its SINR is 18.2 dB after STAP. Note that in this case, the appropriate number of 400 samples, which corresponds to approximately 12 km, is employed to estimate the covariance matrix, and the diagonal loading is not employed here. Based on the peak search to Range-Doppler map, we decided the steering vector which corresponds to the target. Using STAP, the SINR is improved compared to conventional method. The difference between them is 5.4 dB. Therefore, STAP can improve the performance of SINR compared to ordinary method if the appropriate number of samples is applied.
4.2 Relation between the number of samples and SINR after STAP

Fig. 2(a) shows the relation between the number of range samples and SINR. Without STAP, which means ordinary method, SINR is constant since weight vector (6) does not depend on the number of samples.

In case with STAP with no loading, SINR is worse than one without STAP in the large sample region, the number of which is more than 800, which corresponds to approximately 24 km. This implies that the clutter’s characteristic of target’s range bin is different from the far range bins. Thus, the covariance matrix estimated by too large samples does not represent the clutter’s characteristics at target’s range bin.

We have calculated the statistical values about amplitude of Doppler spectrum. As a result, at far range area that is from approximately 66 km to 76 km, the peak of amplitude and average and standard deviation of Doppler frequency was different from ones at near and around target range from 45 km to 66 km. The difference in the peak positions of the amplitude distribution between far range area and around area is 20.4 Hz which corresponds to 3 Doppler bin and the difference of amplitude between them was 1.5 dB. Therefore, the statistical difference makes SINR worse than one after STAP applied 400 optimal sample support.

In the small sample region, the number of which is less than 256, which corresponds to approximately 7.68 km, the SINR performance is poor because $\hat{R}^{-1}$ may be singular or inaccurate matrix. Therefore we introduce the diagonal loading to estimate the covariance matrix, which can improve rank deficiency and find the contribution to the small samples.

In case with the diagonal loading, SINR is better than one without diagonal loading. When the diagonal loading is employed, the factor of which is 3 dB relative to its noise level, 230 samples are the optimal samples to maximize the SINR, which correspond to around 3.45 km area. In comparison with the components of eigenvalue between 230 samples with diagonal loading and 400 samples without diagonal loading, they are nearly similar. It suggests that the components of eigenvalue represent real clutter signal components precisely.
Fig. 2(b) shows the A-scope of the sea clutter and the real target signal when it is applied STAP with 3 dB diagonal loading with 230 samples support. Using the diagonal loading, the SINR is more improved and the difference between the diagonal loading case and the conventional case is 7.8 dB.

According to (5), the greater the number of samples is, the better SINR is achieved when $K \geq N$. If we use samples more than 800, it degrades the SINR performance compared to one of the ordinary method. This results show that IID cannot be always applied in real circumstance.

5 Conclusion

We have used the real sea clutter data observed in S-band at the Sea of Japan in winter to evaluate the SINR performance of STAP, which depends on the number of samples along the range. As a result, with the appropriate number of samples, we have found that STAP can improve the SINR better than ordinary method in the observed data, namely too large samples degrade its SINR. This means in real circumstance IID cannot be always applied. As the result of the above discussion, we show that, in the sea state of 1, the appropriate number of samples is 230. That means samples around only 3.45 km area should be used to carry out STAP for this case. To overcome the degradation of $\Gamma^{-1}$ due to smaller number of samples mentioned above, the diagonal loading was applied. The resultant SINR was improved when 3 dB loading factor was chosen.

We will continue to collect the real sea clutter data about various conditions, for instance other sea state, weather condition, etc. Furthermore, we need to find the optimum samples in STAP according to sea clutter’s condition.

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Rate-distortion-perception tradeoff of variable-length source coding for general information sources

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Abstract: Blau and Michaeli recently introduced a novel concept for inverse problems of signal processing, that is, the perception-distortion tradeoff. We introduce their tradeoff into the rate distortion theory of variable-length lossy source coding in information theory, and clarify the tradeoff among information rate, distortion and perception for general information sources. We also discuss the fixed-length coding with average distortion criterion that was missing in the previous letter.

Keywords: perception-distortion tradeoff, rate-distortion theory, data compression, variable-length coding

Classification: Fundamental Theories for Communications

References


1 Introduction

An inverse problem of signal processing is to reconstruct the original information from its degraded version. It is not limited to image processing, but it often arises in the image processing. When a natural image is reconstructed, the reconstructed image sometimes does not look natural while it is close to the original image by

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a reasonable metric, for example mean squared error. When the reconstructed information is close to the original, it is often believed that it should also look natural.

Blau and Michaeli [1] questioned this unproven belief. In their research [1], they mathematically formulated the naturalness of the reconstructed information by a distance of the probability distributions of the reconstructed information and the original information. The reasoning behind this is that the perceptual quality of a reconstruction method is often evaluated by how often a human observer can distinguish an output of the reconstruction method from natural ones. Such a subjective evaluation can mathematically be modeled as a hypothesis testing [1]. A reconstructed image is more easily distinguished as the variational distance \( \sigma(P_R, P_N) \) increases [1], where \( P_R \) is the probability distribution of the reconstructed information and \( P_N \) is that of the natural one. They regarded the perceptual quality of reconstruction as a distance between \( P_R \) and \( P_N \). The distance between the reconstructed information and the original information is conventionally called as distortion. They discovered that there exists a tradeoff between perceptual quality and distortion, and named it as the perception-distortion tradeoff.

Claude Shannon [2, Chapter 5] initiated the rate-distortion theory in 1950’s. It clarifies the tradeoff between information rate and distortion in the lossy source coding (lossy data compression). The rate-distortion theory has served as a theoretical foundation of image coding for past several decades, as drawing a rate-distortion curve is a common practice in research articles of image coding. Since distortion and perceptual quality are now considered two different things, it is natural to consider a tradeoff among information rate, distortion and perceptual quality. Blau and Michaeli [1] briefly mentioned the rate-distortion theory, but they did not clarify the tradeoff among the three. Then the author [3] clarified the tradeoff among the three for fixed-length coding, but did not clarified variable-length coding, where fixed and variable refer the length of a codeword is fixed or variable, respectively [2, Chapter 5]. The variable-length lossy source coding is practically more important than the fixed-length counterpart because most of image and audio coding methods are variable-length.

The purpose of this letter is to mathematically define the tradeoff of variable-length lossy source coding for general information sources, and to express the tradeoff in terms of information spectral quantities introduced by Han and Verdú [2]. We also discuss the fixed-length coding with average distortion criterion that was missing in the previous letter [3].

Since the length limitation is strict in this journal, citations to the original papers are replaced by those to the textbook [2], and the mathematical proof is a bit compressed. The author begs readers’ kind understanding. The base of log is an arbitrarily fixed real number \( >1 \) unless otherwise stated.

### 2 Preliminaries

The following definitions are borrowed from Han’s textbook [2]. Let

\[
X = \{ X^n = (X_1^{(n)}, \ldots, X_n^{(n)}) \}_{n=1}^\infty
\]
be a general information source, where the alphabet of the random variable $X^n$ is the $n$-th Cartesian product $\mathcal{X}^n$ of some finite alphabet $\mathcal{X}$. For a sequence of real-valued random variables $Z_1, Z_2, \ldots$ we define
\[
\text{p-}\limsup_{n \to \infty} Z_n = \inf \{ \alpha \mid \lim_{n \to \infty} \Pr[Z_n > \alpha] = 0 \}.
\]
For two general information sources $X$ and $Y$ we define
\[
\hat{I}(X;Y) = \text{p-}\limsup_{n \to \infty} \frac{1}{n} \log \frac{P_{X^n,Y^n}(X^n,Y^n)}{P_{X^n}(X^n)P_{Y^n}(Y^n)},
\]
\[
H_K(X) = \limsup_{n \to \infty} \frac{1}{n} H_K(X^n),
\]
\[
F_X(R) = \limsup_{n \to \infty} \Pr \left[ \frac{1}{n} \log \frac{1}{P_{X^n}(X^n)} \geq R \right],
\]
where $H_K(X^n)$ is the Shannon entropy of $X^n$ in $\log K$.

For two distributions $P$ and $Q$ on an alphabet $\mathcal{X}$, we define the variational distance $\sigma(P,Q)$ as $\sum_{x \in \mathcal{X}} |P(x) - Q(x)|/2$. In the rate-distortion theory, we usually assume a reconstruction alphabet different from a source alphabet. In order to consider the distribution similarity of reconstruction, in this letter we assume $\mathcal{X}^n$ as both source and reconstruction alphabets.

### 3 Variable-length source coding

An encoder of length $n$ is a stochastic mapping $f_n: \mathcal{X}^n \to \mathcal{U}^*$, where $\mathcal{U} = \{1, \ldots, K\}$ and $\mathcal{U}^*$ is the set of finite-length sequences over $\mathcal{U}$. By stochastic we mean that the encoder output $f_n(x^n)$ is probabilistic with a fixed input $x^n \in \mathcal{X}^n$. The corresponding decoder of length $n$ is a deterministic mapping $g_n: \mathcal{U}^* \to \mathcal{X}^n$. We denote by $|f_n(x^n)|$ the (random variable of) length of sequence $f_n(x^n) \in \mathcal{U}^*$ for $x^n \in \mathcal{X}^n$. We denote by $\delta_n: \mathcal{X}^n \times \mathcal{X}^n \to [0,\infty)$ a general distortion function.

#### 3.1 Average distortion criterion

**Definition 1** A triple $(R,D,S)$ is said to be va-achievable if there exists a sequence of encoder and decoder $(f_n,g_n)$ such that
\[
\limsup_{n \to \infty} \frac{\mathbb{E}[(|f_n(X^n)|)]}{n} \leq R, \tag{1}
\]
\[
\limsup_{n \to \infty} \frac{1}{n} \mathbb{E}[(\delta_n(X^n,g_n(f_n(X^n))))] \leq D, \tag{2}
\]
\[
\limsup_{n \to \infty} \sigma(P_{g_n(f_n(X^n))},P_{X^n}) \leq S. \tag{3}
\]

Define the function $R_{va}(D,S)$ by
\[
R_{va}(D,S) = \inf \{ R \mid (R,D,S) \text{ is va-achievable} \}.
\]

**Theorem 2**
\[
R_{va}(D,S) = \inf_Y H_K(Y)
\]
where the infimum is taken with respect to all general information sources $Y$ satisfying
\[
\limsup_{n \to \infty} \frac{1}{n} E[\delta_n(X^n, Y^n)] \leq D,
\]
\[
\limsup_{n \to \infty} \sigma(P_{Y^n}, P_{X^n}) \leq S.
\]

**Proof.** Let a pair of encoder \( f_n \) and decoder \( g_n \) satisfies Eqs. (1)–(3). Let \( Y^n = g_n(f_n(X^n)) \), and define the general information source \( Y \) from \( Y^n \). We immediately see that \( Y \) satisfies Eqs. (4) and (5). By the same argument as [2, p. 349] we immediately see

\[
\limsup_{n \to \infty} \frac{1}{n} E[\log |f_n(X^n)|] \geq H_K(Y).
\]

On the other hand, suppose that a general information source \( Y \) satisfies Eqs. (4) and (5). Let \( f'_n \) and \( g'_n \) be a lossless variable-length encoder and its decoder [2, Section 1.7] for \( Y \) such that \( Y^n = g'_n(f'_n(Y^n)) \) and

\[
\limsup_{n \to \infty} \frac{1}{n} E[\log |f'_n(Y^n)|] = H_K(Y).
\]

For a given information sequence \( x^n \in X^n \), the encoder randomly chooses \( y^n \in Y^n \) according to the conditional distribution \( P_{Y^n|X^n}(\cdot|x^n) \), and define the codeword as \( f'_n(y^n) \). The decoding result is \( y^n = g'_n(f'_n(y^n)) \). Since the probability distribution of decoding result \( g'_n(f'_n(y^n)) \) is \( P_{Y^n} \), we see that the constructed encoder and decoder satisfy Eqs. (2) and (3). \( \square \)

### 3.2 Maximum distortion criterion

**Definition 3** A triple \( (R, D, S) \) is said to be vm-achievable if there exists a sequence of encoder and decoder \( (f_n, g_n) \) such that

\[
\limsup_{n \to \infty} \frac{1}{n} E[\log |f_n(X^n)|] \leq R,
\]

\[
p-\limsup_{n \to \infty} \frac{1}{n} \delta_n(X^n, g_n(f_n(X^n))) \leq D,
\]

\[
\limsup_{n \to \infty} \sigma(P_{Y^n, f_n(X^n)}, P_{X^n}) \leq S.
\]

Define the function \( R_{vm}(D, S) \) by

\[
R_{vm}(D, S) = \inf \{ R \mid (R, D, S) \text{ is vm-achievable} \}.
\]

**Theorem 4**

\[
R_{vm}(D, S) = \inf Y H_K(Y)
\]

where the infimum is taken with respect to all general information sources \( Y \) satisfying Eq. (5) and

\[
p-\limsup_{n \to \infty} \frac{1}{n} \delta_n(X^n, Y^n) \leq D.
\]

**Proof.** The proof is almost the verbatim copy of that of Theorem 2 and is omitted. \( \square \)
Remark 5 The tradeoff for variable-length coding with the average distortion criterion and without the perception criterion was also determined by using stochastic encoders [2, Section 5.7], but with the maximum distortion criterion without the perception criterion, only the deterministic encoders were sufficient to clarify the tradeoff [2, Section 5.6]. It is not clear at present whether or not we can remove the randomness from encoders in Theorem 4.

4 Fixed-length coding with the average distortion criterion
In this section we state the tradeoff for fixed-length coding with the average distortion criterion, because it has never been stated elsewhere. The proof is almost the same as [3]. Note that the definition of encoder will be different from Section 3 and that an assumption on the distortion $\delta_n$ will be added.

An encoder of length $n$ is a deterministic mapping $f_n: \mathcal{X}^n \to \{1, \ldots, M_n\}$, and the corresponding decoder of length $n$ is a deterministic mapping $g_n: \{1, \ldots, M_n\} \to \mathcal{X}^n$. We require an additional assumption that $\delta_n(x^n, x^n) = 0$ for all $n$ and $x^n \in \mathcal{X}^n$.

Definition 6 A triple $(R, D, S)$ is said to be fa-achievable if there exists a sequence of encoder and decoder $(f_n, g_n)$ such that
\[
\limsup_{n \to \infty} \frac{\log M_n}{n} \leq R,
\]
\[
\limsup_{n \to \infty} \frac{1}{n} \mathbb{E}[\delta_n(\mathcal{X}^n, g_n(f_n(\mathcal{X}^n)))] \leq D,
\]
\[
\limsup_{n \to \infty} \sigma(P_{g_n(f_n(\mathcal{X}^n))}, P_{\mathcal{X}^n}) \leq S.
\]

Define the function $R_{fa}(D, S)$ by
\[
R_{fa}(D, S) = \inf\{R \mid (R, D, S) \text{ is fa-achievable}\}.
\]

Theorem 7
\[
R_{fa}(D, S) = \max \left\{ \inf_{\mathbf{Y}} \hat{I}(\mathbf{X}; \mathbf{Y}), \inf\{R \mid F_X(R) \leq S\} \right\}
\]
where the infimum is taken with respect to all general information sources $\mathbf{Y}$ satisfying
\[
\limsup_{n \to \infty} \frac{1}{n} \mathbb{E}[\delta_n(\mathcal{X}^n, Y^n)] \leq D.
\]

Proof. Proof is almost the verbatim copy of that of [3].